# NUMERICAL STUDY OF EJECTION OF MATERIAL 

## FROM A SOLID SURFACE BY INTENSE RADIATION

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The ejection process is described gas-dynamically for the case of normal incidence ona surface by a flow of radiation. A complete equation system is written down which is solved numerically. In this situation, the symmetry of the ejection offers an opportunity to eliminate one of the variables and makes it possible to carry out the calculation over time in atwo-dimensional physical space. Absorption of the radiation energy by the solid is taken into account by the introduction of a discontinuity; the gas-dynamic quantities on both sides of the discontinuity are related by conservation laws and, at zero time, by the Jouguet rule. The gas is assumed to be ideal. The results of the calculation are presentedin the form of curves. Conditions at the discontinuity which replaces the region of heating adjacent to the solid are discussed.

For presently existing radiation pulse lengths, the nature of the processes arising through interaction of radiation with the surface is determined mainly by the value of the radiation flux density $q$, which can vary over wide limits [1-4].

Low densities q correspond to conditions where there is no marked vaporization of the heated surface, but emission of electrons and ions occurs. For somewhat greater flux densities, disintegration of the surface begins and a vapor cloud - a flare - appears. If the radiation flux density is not so great that the flare is significantly ionized during the time of the pulse and blocks access of the radiation to the vaporized surface, separate investigation of the gas-dynamic and optical problems is permissible for such $q$. The distribution of density, vapor temperature, and degree of condensation are determined from a solution of the problem of vapor ejection from the disintegrating surface. From them, one can calculate the absorption and scattering of radiation by breakdown products and thereby correct the value for the light flux affecting the material by considering absorption in the ejected material. For a constant value of $q$ with vapor expansion into a vacuum, the characteristics of the resultant self-similar motion [5, 6] can then be determined.

For large radiation flux densities corresponding, for example, to giant laser pulses and produced by focusing a flow of these pulses on the surface of a sample, ionization of the flare begins to play a fundamental role in the ejection process because the effect of thermal conductivity is reduced for large $q$, the temperature and degree of ionization increase leading to an increase in the absorption coefficient and screening of the surface. Nonuniformity in the ejection of radiation-absorbing plasma which then occurs is important because the expansion proceeds both in the direction of incident radiation and in directions parallel to the surface of the sample $[7,8]$.

In the general case, the problem of radiation interaction with a surface contains three spatial variables and a time coordinate. In this paper, normal incidence of radiation on the surface is considered, which makes it possible to eliminate one of the spatial variables. We select a cylindrical coordinate system $r, \varphi$, and $z$, with the $z$ axis extending from the center of the focal spot in a direction opposite to that of the radiation flow (Fig. 1).

Because of the symmetry of ejection with respect to the $z$ axis, the equations do not contain the variable $\varphi$ and the problem is solved in a plane $\varphi=$ const. In such a formulation, one can compare the computed

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Fig. 1


Fig. 2
results directly with experimental data since lateral expansion of the flare is taken into consideration.

We shall consider the gas-dynamic approximation valid in the dense region of the flare with the consequence the process can be described by the equation system

$$
\begin{gather*}
\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v} \nabla) \mathbf{v}+\frac{\nabla p}{\rho}=0, \quad \frac{\partial \rho}{\partial t}+\nabla(\rho \mathbf{v})=0  \tag{1}\\
\frac{\partial}{\partial t}\left(\rho \varepsilon+\frac{\rho v^{2}}{2}\right)+\nabla\left[\rho \mathbf{v}\left(\varepsilon+\frac{p}{\rho}+\frac{v^{2}}{2}\right)+q\right]=0
\end{gather*}
$$

where v is velocity, p is pressure, $\rho$ is density, $\varepsilon=\mathrm{p} /[\rho(x-1)]+\Omega$, and $x$ is the Poisson adiabat constant. By $\Omega$ is meant the sum of the heat of fusion, the heat of vaporization, and the heat to get to the vaporization temperature.

To determine the temperature, it is assumed there exists an equation of state

$$
\begin{equation*}
p=R \rho T / \mu \tag{2}
\end{equation*}
$$

where $R$ is the universal gas constant, $\mu$ is the molecular weight, and $T$ is temperature.

We assume the motion occurs in the region abcded ${ }^{\prime} c^{\prime} b^{\prime}$ (Fig. 1) on the boundaries of which the following conditions are satisfied.

1. On $b^{\prime} a b$ or $b_{1}^{\prime} b_{1}$, i.e., on a discontinuity, the following relations are valid:

$$
\begin{align*}
& \rho_{1}\left(D-p_{1}\right)=\rho_{v} D, p_{1}-p_{0}=\rho_{0} v_{1} D \\
& -\rho_{0} D\left(\varepsilon_{1}-\varepsilon_{0}+v_{1}^{2} / 2\right)+p_{1} v_{1}=q \tag{3}
\end{align*}
$$

Here, $D$ is the velocity of the vaporization wave. These relations are insufficient for the determination of $p_{1}, \rho_{1}, v_{1}, D$, and $p_{0}$ by means of the quantities $\rho_{0}$ and $v_{0}=0$ to the left of the discontinuity $b^{\prime} a b$. It is therefore necessary to assign an initial temperature $T_{0}$ in the flare and to use the equation of state (2). In addition, it is assumed that at $t=0$ the Jouguet relation

$$
\begin{equation*}
v_{1}-\sqrt{x p_{1 / p_{1}}}=D \tag{4}
\end{equation*}
$$

which is subsequently not used (for $t>0$ ) is satisfied, and extrapolation of the velocity $v$ from the flare to an arbitrary discontinuity is made for the numerical solution of Eq. (1).

Generally speaking, the quantities $x$ and $\mu$ are variables, but the temperature does not vary greatly near the surface of the solid material because "cold" material flows in from the discontinuity and the variation of $x$ and $\mu$ cannot be taken into account. Temperature at a discontinuity at subsequent times will be assumed to be equal to the temperature of the neighboring layer.
2. The surfaces $b c, c d, d^{\prime} c^{\prime}$, and $c^{\prime} b^{\prime}$ are smooth, solid walls.
3. On ded', the pressure is zero, which is achieved by linear extrapolation of velocity and pressure into a vacuum. Density is determined from pressure and the temperature of the neighboring layer.

The system (2)-(4) of boundary conditions has a unique real solution

$$
v_{1}=\left[\sqrt[3]{-\alpha / 2+\sqrt{Q}}+\sqrt[3]{-\alpha / 2-\sqrt{Q}}-(2-x) \sqrt{T_{0} / x / 3}\right] / \sqrt{x-1}
$$

where

$$
\begin{gather*}
\alpha=2\left\{q / \rho_{0}+\sqrt{T_{0}^{3}(x-1) / x}(2-x)^{2}\left[(2-x)(x-1) / 9 x-1-3 x /(2-x)^{2}\right] / 3\right\}  \tag{5}\\
\beta=T_{0}\left(4-x^{2}\right)(x+1) / 3 x(x-1), Q=(\beta / 3)^{3}+(\alpha / 2)^{2} \\
D=v_{1}-\sqrt{x T_{0}^{\prime} /(x-1)}, \rho_{1}=\rho_{0} D /\left(D-v_{1}\right) . \\
p_{1}=\rho T_{0}, p_{0}=p_{1}+\rho_{1} v_{1} \sqrt{x T_{0} /(x-1)}
\end{gather*}
$$

For analysis of the boundary conditions, it is convenient to transform Eqs. (2)-(4) slightly

$$
\begin{gather*}
\rho_{1}\left(D-v_{1}\right)=\rho_{0} D, \quad x p_{1}=\rho_{0} v_{1} D  \tag{6}\\
-\rho_{0} D\left(\varepsilon_{1}-\varepsilon_{0}+v_{1}^{2} / 2\right)+p_{1} v_{1}=q, \quad v_{1}-\sqrt{\chi \rho_{1} / \rho_{\mathrm{x}}}=D
\end{gather*}
$$

where

$$
x=1-p_{0} / p_{1}
$$

Here, $p_{1}, \rho_{1}, v_{1}$, and $D$ are unknowns, and $x, \rho_{0}, v_{0}$ are given. As the solution of the system (2)-(4) in the form (5) shows, the quantity $x$ is weakly dependent on $q$ for $x=1.2$ and varies from $x=0.6$ for $q=2 \cdot 10^{7}$ $\mathrm{W} / \mathrm{cm}^{2}$ to $\mathrm{x}=0.56$ for $\mathrm{q}=2 \cdot 10^{9} \mathrm{~W} / \mathrm{cm}^{2}$. For an x determined from the solution of the system (2)-(4), the solutions of (6) and (2)-(4) agree, and one can limit oneself to an investigation of the system (6) for a given $x$.

System (6) has the unique real solution

$$
\begin{equation*}
v_{1}=x\left\{q /\left[\rho_{0}\left(-x^{2}+2 x-\frac{2 x}{x-1}\right)(x+x)\right]\right\}^{1 / 3} F\left(\Omega, p_{0}, q, x, x\right) \tag{7}
\end{equation*}
$$

where $\mathbf{F}$ is some function of the arguments given, and

$$
\begin{equation*}
\rho_{1}=\rho_{0}(1+x / x), \quad p_{1}=v_{1}^{2} \rho_{0}(x+x) / x^{2}, \quad D=v_{1}(1+x / x) \tag{8}
\end{equation*}
$$

From the equations for $\rho_{1}$ and $p_{1}$ and from the definition of $x$, it follows that $-x<x<1$. The function $F$ varies monotonically from 0 to $\sqrt[3]{2}$ as $q$ increases from 0 to $\infty$. For example, when $q \approx 10^{10} \mathrm{~W} / \mathrm{cm}^{2}, \mathrm{~F}=1$; when $q=10^{12} \mathrm{~W} / \mathrm{cm}^{2}, F$ can be considered independent of both $q$ and $\Omega$. This means that in the region of large radiation flux densities, the radiation energy absorbed in a discontinuity goes entirely into kinetic energy of the heated layer.

The functions $v_{1}, \rho_{1}, p_{1}$, and $D$ obtained from the solution of the boundary conditions (3)-(4) are shown as curves in Fig. 2. A shock wave can be propagated in the cold material ahead of the discontinuity [1, 9]. To determine the conditions for its existence, one can use relations between the sound velocities con both sides of the discontinuity and the velocity of the discontinuity itself.

We have

$$
c_{1}^{2}=x p_{1} / \rho_{1}, \quad c_{0}^{2}=x p_{0} / \rho_{0}
$$

Using these relations, the expression of $\rho_{1}$ through $\rho_{0}$ in Eq. (8), and the definition of $x$, we find there are two different modes of flow, in one of which propagation of the shock wave ahead of the "vaporization wave" is possible. In correspondence with this, three regions $A, B$, and $C$ can be distinguished in Fig. 2 such that the conditions

$$
|D|<c_{0}<c_{1},|D|<c_{1}<c_{0},|D|>c_{1}>c_{0} .
$$

are satisfied in the respective regions.
Regions A and B correspond to a mode of flow where a shock wave ahead of the discontinuity is possible - "burning," and in C the shock wave coincides with the discontinuity - "detonation." In region $C$, the shock wave coincides with $b^{\prime} a b$ and travels along uncompressed material, i.e., $p_{0}=0, x=1$, and $|D|=\max |D|$. Consequently, only a single point with maximum detonation velocity is realized in the entire region $C$ - the point $\gamma$.

The equation system (1), rewritten in cylindrical coordinates, is solved numerically. For the assignment of initial distributions by means of conditions (2)-(4), the material is taken to be graphite. The initial density $\rho_{0}=2.3 \mathrm{~g} / \mathrm{cm}^{3}$ and the dimension $\mathrm{ab}=0.015 \mathrm{~cm}$ [8].

At zero time, the transverse velocity $u=0$ and the longitudinal velocity $v=$ const. The initial distribution of $p$ was chosen in the form of linear functions of $r$ and $z$ in the region abde.

Starting from $z=0$, the pressure falls for all $r$. It is constant along $r$ for $0 \leq r \leq r_{0} \leq r_{b}$ and decreases linearly for $r_{0} \leq r \leq r b$. Such a distribution is necessary in order to start the calculation. The initial density is given in Eq. (2) for fixed initial temperature $\mathrm{T}_{0} \approx 10^{4}{ }^{\circ} \mathrm{K}$ and the already known pressure. The exact assignment plays no great part because the first ionization potential of graphite, up to the temperature of which the calculation is carried out, is sufficiently high $\left(1.2 \cdot 10^{5}{ }^{\circ} \mathrm{K}\right)$ and the magnitude of the flare temperature will be an order of magnitude greater than $T_{0}$ before the beginning of intense ionization.


Fig. 3


Fig. 5


Fig. 7


Fig. 9


Fig. 4


Fig. 6


Fig. 8


Fig. 10

Laser radiation is incident on the solid surface of the graphite [8]; one can the refore assume that there is no line absorption for the temperatures $T$ under consideration ( $\mathrm{T}_{0} \leq \mathrm{T} \leq 8 \cdot 10^{4}{ }^{\circ} \mathrm{K}$ ) and the absorption coefficient K is given by the Kramers-Unsold formula

$$
K=0.96 \cdot 10^{-7} N Z^{2} T^{-2} A^{-3} \exp \left(A-A_{1}\right)
$$

where $N$ is the number of atoms, $Z$ is the atomic number, $T$ is the temperature, $A_{1}=I / k T, I$ is the ionization potential, $k$ is the Boltzmann constant, $A=h \nu / k T$, and $h \nu$ is the quantum energy [10]. The effects of ionization nonequilibrium [11] are not taken into account in this case.

An implicit difference scheme was used in the calculations in which the time derivatives $\partial f / \partial t$ were replaced by the difference $\left(f^{n+1}-f^{n}\right) / \Delta t$ and the derivatives with respect to coordinates $\partial f / \partial y$ were replaced by $\left(f_{i+1}^{k+1}-f_{i-1}^{k+1}\right) / 2 \Delta y$. An iteration method was used in the solution.

Such a scheme is little different from the "conversion" scheme used in [12]. Its stability was evaluated by the Neumann method. For this, all gas-dynamic quantities $f_{i}(y, t)$ were represented in the form $f_{i}+f_{i}^{*}(y, t)$, where $f_{i}=$ const and $f^{*}(y, t)$ is a variation which is considered a small quantity of first order [13]. Then the difference scheme for $f_{i}{ }^{*}(y, t)$ reduces to the form $G f^{\prime} n+1(y)=f^{*} n$, where $G$ is a transformation matrix. For stability, it is necessary that the eigenvalues $\lambda$ of the transformation matrix Gall be greater than 1. We have

$$
\lambda_{1} \geqslant 1+i v \Delta t / \Delta y \sin n \Delta y \geqslant 1
$$

where, for simplicity, we assume $\Delta \mathrm{z}=\Delta \mathrm{r} \equiv \Delta \mathrm{y}$. To obtain $\lambda_{2}, \lambda_{3}$, and $\lambda_{4}$, we use the smallness of the quantity $l=v \Delta t / \Delta y$, which is no more than $10^{-2}$ in this problem and neglect terms of order $l^{2}$. Then

$$
\lambda_{2,3} \geqslant 1+i l \sin n \Delta y \geqslant 1, \quad \lambda_{4} \geqslant 1+u \frac{\Delta t}{r}(1+x)+i l \sin n \Delta y-\delta
$$

where $u$ is the velocity in the direction $r, r \geq \Delta y$. The quantity $\delta$ is proportional to $q \Delta t$ and arises because of heating of the gas by radiation. It is negative for

$$
p F \Delta y \sum_{m} \exp \left(-H p_{m} / p_{m}+i m n \Delta y\right)>1
$$

where the summation is carried out over points along the $z$ axis at some $r ; H \approx 10^{-2}, F \approx 200$. For $q=10^{9}$ $\mathrm{W} / \mathrm{cm}^{2}, \mathrm{p} \sim 5 \cdot 10^{-2}, \mathrm{p}_{\mathrm{m}} / \rho_{\mathrm{m}} \sim 10^{-3}$, and the stability condition is satisfied.

For small $q$, the quantity $\Delta$ is small because $q$ appears in it as a multiplying factor. Thus the eigenvalues of the inverse of the transformation matrix are greater than one and the scheme is stable.

Results of the calculations are shown in Figs. 3-10. In solving the boundary equations (2)-(4), it was observed that for a change in $q$ from $q_{\min }=2 \cdot 10^{7} \mathrm{~W} / \mathrm{cm}^{2}$ to $2 \cdot 10^{9} \mathrm{~W} / \mathrm{cm}^{2}$ (in dimensionless units, from 0.2 to 20 ), the dimensionless velocity $v_{1}$ varied from 0.2881 to 0.2786 , i.e., practically remained constant. There was also practically no time dependence for $v_{1}$ for variation of $q$ within the limits mentioned and times of the order of $10^{-8} \mathrm{sec}$. This made it possible to limit examples of the time dependence of v to $\mathrm{q}=5 \cdot 10^{8} \mathrm{~W} /$ $\mathrm{cm}^{2}$ only (curve 1 in Fig. 3 refers to the time $\mathrm{t}=4 \mathrm{nsec}$, curve 2 to the time $\mathrm{t}=8 \mathrm{nsec}$ ).

The boundary values $p_{1}$ and $\rho_{1}$ are practically proportional to the quantity $q$ and in Figs. 4 and 5 , curves are given which for convenience use the same scaling of the quantities $p$ and $\rho$ ( $p$ and $\rho$ are multiplied by the ratio of the minimum $q$ considered, $q_{\text {min }}$, to the $q$ corresponding to a given $p$ or $\rho$ ), i.e., $p_{*}=\left(q_{m i n} / q\right) p, p_{*}=$ $\left(q_{\min } / q\right) \rho$. The curves 1 correspond to $q=5, t=4 \mathrm{nsec}$, the curves 2 to $q=5, t=8 \mathrm{nsec}$, and the curves 3 to $q=10, t=4 \mathrm{nsec}$; the dashed line indicates the initial pressure distribution. The drop in the values of $p$ and $\rho$ at the point $\mathrm{z}=0$ is somewhat artificial and results from a decrease in radiation flux density at the boundary with the solid because of an increase of radiation absorption in the flare with time.

Figure 6 shows the flare temperature distribution along $z$ for $r=0$ and $q=10^{9} \mathrm{~W} / \mathrm{cm}^{2} 4$ nsec after the beginning of radiation effects. Curve 1 was obtained for 10 initial points along $z$, and curve 2 for 16 points. The other parameters remained unchanged. Rapid heating of the leading edge of the flare is observed - a "spike" - which leads to the appearance of shock waves. The dashed line shows the temperature for superposition of the leading edges in these two cases. Figures 7 and 8 show the pressure distribution and lateral velocity $u$ along the coordinate $r$. The dashed line indicates the initial pressure, the initial velocity $u$ is zero, and the parameters $q$ and $t$ are the same as in Figs. 4 and 5. Figure 9 shows the variation in temperature of the leading edge of the flare as a function of time for $q=3.6 \cdot 10^{9} \mathrm{~W} / \mathrm{cm}^{2}$.

The variation of the parameter ( -x ) (solid lines) and of the quantity $j=x$ (dashed lines) from Eq. (4), which is now calculated from $v_{1}, p_{1}, p_{1}$, and $D$, is shown in Fig. 10 for three values of $q(1-q=2,2-q=10$, $3-q=20$ ). The quantity $j$ decreases with time which leads to subsonic outflow from the discontinuity for an initial motion ( $t=0$ ) close to sonic ( $|\mathrm{D}| \ll v$, and their difference at $t=0$ equals the velocity of sound). An increase in $x$ leads to a change in boundary values in accordance with the diagram in Fig. 2.

Viscosity was not included in the calculational scheme; therefore it was adjusted to produce smooth solutions only and the calculations were carried forward to the time of appearance of shock waves, which depended on the value of the initial temperature $\mathrm{T}_{0}$. Their formation resulted from the spike and from the lack of equilbrium observed during the calculation in the distribution of radiation flux density along the coordinate $r$ at the boundary of the solid.

It should be noted that the appearance of shock waves, nonequilibrium conditions, and other phenomena which accompany the absorption of intense radiation greatly complicate the investigation of the problem. It therefore appears reasonable to solve the problem of prolonged ejection by an approximation of the absorption coefficient with the power function which is valid for multiple ionization. In such a formulation, one can also take into account fast phenomena like a spike as well as nonequilibrium conditions of processes associated with radiation absorption, and compare the results obtained with self-similarity solutions $[6,14,15]$ and with numerical solutions [16].

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